Product of Sum (POS)

A canonical product of sum is a boolean expression that entirely consists of maxterms. The Boolean function F is defined on two variables X and Y. The X and Y are the inputs of the boolean function F whose output is true when only one of the inputs is set to true. The truth table for Boolean expression F is as follows:

Inputs		Output
X	**Y**	**F**
0	0	0
0	1	1
1	0	1
1	1	0

In our minterm and maxterm section, we learned about how we can form the maxterm from the variable's value. A column will be added for the maxterm in the above table. The complement of the variables is taken whose value is 0, and the variables whose value is 1 will remain the same.

Inputs		Output	Minterm
X	**Y**	**F**	**M**
0	0	0	X'+Y'
0	1	1	X'+Y
1	0	1	X+Y'
1	1	1	X+Y

Converting Product of Sum (POS) to shorthand notation

The process of converting POS form to shorthand notation is the same as the process of finding shorthand notation for maxterms. There are the following steps used to find the shorthand notation of the given POS expression.

- Write the given POS expression.

- Find the shorthand notation of all the maxterms.
- Replace the minterms with their shorthand notations in the given expression.

Example: F = (X'+Y').(X+Y)

1. Firstly, we will write the POS expression:

F = (X'+Y').(X+Y)

2. Now, we will find the shorthand notations of the maxterms X'+Y' and X+Y.

X'+Y' = $(00)_2$ = M_0

X+Y = $(11)_2$ = M_3

3. In the end, we will replace all the minterms with their shorthand notations:

F=$M_0.M_3$

Converting shorthand notation to POS expression

The process of converting shorthand notation to POS is the reverse process of converting POS expression to shorthand notation. Let's see an example to understand this conversion.

Example:

Let us assume that we have a boolean function F, defined on two variables X and Y. The maxterms for the function F are expressed as shorthand notation is as follows:

F=∏(1,2,3)

Now, from this expression, we find the POS expression. The Boolean function F has two input variables X and Y and the output of F=0 for M1, M2, and M3, i.e., 1^{st}, 2^{nd}, and 3^{rd} combinations. So,

F=∏(1,2,3)

F= M1.M2.M3

F= 01.10.11

Sum of product(SOP)

A canonical sum of products is a boolean expression that entirely consists of minterms. The Boolean function F is defined on two variables X and Y. The X and Y are the inputs of the boolean function F whose output is true when any one of the inputs is set to true. The truth table for Boolean expression F is as follows:

Inputs		**Output**
X	**Y**	**F**
0	0	0
0	1	1
1	0	1
1	1	1

In our previous section, we learned about how we can form the minterm from the variable's value. Now, a column will be added for the minterm in the above table. The complement of the variables is taken whose value is 0, and the variables whose value is 1 will remain the same.

Inputs		**Output**	**Minterm**
X	**Y**	**F**	**M**
0	0	0	X'Y'
0	1	1	X'Y
1	0	1	XY'
1	1	1	XY

Now, we will add all the minterms for which the output is true to find the desired canonical SOP(Sum of Product) expression.

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F=X' Y+XY'+XY
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Converting Sum of Products (SOP) to shorthand notation

The process of converting SOP form to shorthand notation is the same as the process of finding shorthand notation for minterms. There are the following steps to find the shorthand notation of the given SOP expression.

- Write the given SOP expression.

- Find the shorthand notation of all the minterms.
- Replace the minterms with their shorthand notations in the given expression.

Example: F = X'Y+XY'+XY

1. Firstly, we write the SOP expression:

F = X'Y+XY'+XY

2. Now, we find the shorthand notations of the minterms X'Y, XY', and XY.

$X'Y = (01)_2 = m_1$
$XY' = (10)_2 = m_2$
$XY = (11)_2 = m_3$

3. In the end, we replace all the minterms with their shorthand notations:

F=m1+m2+m3

Converting shorthand notation to SOP expression

The process of converting shorthand notation to SOP is the reverse process of converting SOP expression to shorthand notation. Let's see an example to understand this conversion.

Example:

Let us assume that we have a boolean function F, which defined on two variables X and Y. The minterms for the function F are expressed as shorthand notation is as follows:

$F=\sum(1,2,3)$

Now, from this expression, we will find the SOP expression. The Boolean function F has two input variables X and y and the output of F=1 for m1, m2, and m3, i.e., 1st, 2nd, and 3rd combinations. So,

$F=\sum(1,2,3)$
F= m1 + m2 + m3
F= 01 + 10 + 11

Now, we replace zeros with either X' or Y' and ones with either X or Y. Simply, the complement variable is used when the variable value is 1 otherwise the non-complement variable is used.

$F = \sum(1,2,3)$

$F=01+10+11$

$F= A'B + AB' + AB$